647510 **BSCmaMJ208x-4C** **Seat No :________**

B.Sc. Semester - 4 (NEP-2020) Examination
March/April-2026 (As Per Syllabus Year June-2025)
MATH: Linear Algebra-2 (Major-8)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any TWO: (10)

(1) Define: Symmetric Linear Transformation.
Prove that the eigen vectors corresponding to different eigen values of a symmetric linear transformation defined on a real inner product space are perpendicular.

(2) Define : Orthogonal Complementary Subspace.
Prove that $W \cap W^{\perp} = \{\theta\}$ for any subspace W of an inner product space V (where θ is the zero vector of V.

(3) State and prove the spectral theorem.

Que.2 Answer any TWO: (10)

(1) Prove that the set of all bilinear forms on the vector space $\mathbb{R}^n$ is a vector space over $\mathbb{R}$ with respect to the addition and scalar multiplication of bilinear forms.

(2) If ϕ be a bilinear form on the real vector space $\mathbb{R}^n$, then prove that there exists unique matrix $A_{n\times n}$ such that $\phi(x, y) = xAy^T, \forall x, y \in \mathbb{R}^n$.

(3) Explain the relationship between a linear transformation and a bilinear form.

Que.3 Answer any TWO: (10)

(1) Define: Quadratic Form
Discuss the method of Reduction of a Quadratic Form to its Canonical Form.

(2) Describe the properties of Quadratic Forms which remain invariant under a change of coordinates.

(3) Find the canonical form of the quadratic form
$Q(x) = {x_1}^2 + {x_3}^2 + 6x_2x_3 - 2x_3x_1 + 4x_1x_2$ on $\mathbb{R}^3$. where $x = (x_1, x_2, x_3) \in \mathbb{R}^3$.

Que.4 Answer any TWO: (10)

(1) Define: Orthogonal linear transformation
Prove that the composition of two orthogonal linear transformations is again an orthogonal linear transformation.

(2) What is an Affine Linear Transformation?
How does it differ from a standard linear transformation? Explain.

(3) If $H = f \circ T: V \to V$ be any affine transformation on the vector space V
where $f(x) = x + a, \forall x \in V$ and $T: V \to V$ is a linear transformation,
then prove that T is non-singular if and only if H is so.
Also, show that $H^{-1} = g \circ T^{-1}$, where $g(x) = x - T^{-1}(a), \forall x \in V$.

Que.5 Answer any TWO: (10)

(1) Find the diagonal matrix similar to the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

(2) Explain: Congruent Quadratic Forms and the relation of congruence for matrices.

(3) Let $H: \mathbb{R}^2 \to \mathbb{R}^2, H(x, y) = (3x + 6y + 2, 3y - 4), \forall (x, y) \in \mathbb{R}^2$. Then, show that H is an affine transformation and find H^{-1} if exists.

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